

WEB APPENDIX

Sarstedt, M. & Mooi, E. (2019). *A concise guide to market research. The process, data, and methods using SPSS* (3rd ed.). Heidelberg: Springer.

χ^2 -tests

The χ^2 -test (pronounced as *chi square*) is commonly used when examining nominal and ordinal variables. There are two types of χ^2 -tests involving nominal data (note that they can also be applied to ordinal data):

- 1.) The *one-sample χ^2 -test* (also called the χ^2 goodness-of-fit test) is a nonparametric test, which tests whether the number of cases observed for a distribution, align with a set of expected values.
- 2.) The *χ^2 -test for independence* is used to determine whether two nominal (or ordinal) variables are related.

An example of the one-sample χ^2 -test: Suppose a mobile phone producer plans to launch a new smartphone on the market, but is not sure which color to use. 200 people are chosen from the target group, of which 40 indicate “black,” 84 “silver,” and 76 “red.” If we want to establish whether or not these preferences differ significantly from what could be expected a priori, this can be determined by means of a one-sample χ^2 -test. In the simplest case, we would expect all three colors to be equally popular, which means that we expect 66.67 respondents to choose each of these colors. Comparing the sample values with these expected values, we conclude that there is a difference between the two. The question is whether this difference is attributable to the sample variation, or whether the difference holds for the population. If the latter is the case, we could conclude that silver is the preferred smartphone color in the population. This is what we are going to explore by using the one-sample χ^2 -test, whose null hypothesis states that there is no difference between the expected and observed values.

We can test this null hypothesis by using the χ^2 -test statistic, which is calculated by collecting observed values for each of the categories and examining the differences between the observed and expected values (IBM 2011):

$$\chi^2 = \sum_{i=1}^k \frac{(h_i - \tilde{h}_i)^2}{\tilde{h}_i},$$

where h_i is the observed value for category i and \tilde{h}_i is the expected value for the i^{th} category, and k is the total number of categories. In other words, the χ^2 -test statistic is the sum of the squared differences between the observed and expected values, divided by the expected values.

As mentioned earlier, we expect there to be $200/3=66.67$ respondents who prefer each of the three colors, which yields the following:

$$\chi^2 = \frac{(40 - 66.67)^2}{66.67} + \frac{(84 - 66.67)^2}{66.67} + \frac{(76 - 66.67)^2}{66.67} = 10.67 + 4.50 + 1.31 = 16.48$$

The test value is not directly interpretable, but must be compared to the critical value obtained from the χ^2 -statistic with $k-1$ (in our example $3 - 1 = 2$) degrees of freedom—see Table A3 in the [Web Appendix \(→ Downloads → Additional Material\)](#). As the test statistic value (16.48) is much higher than the critical value (5.991; $\alpha=0.05$), we can reject the null hypothesis and, thus, conclude that the preferences differ significantly from what could be expected.¹ In this example, we assumed that each category has the same expected frequency (66.67).

In the previous example, we considered only one sample, but researchers are frequently interested in evaluating whether there is a significant relationship between two nominal variables. Suppose we examined the survey described above further by distinguishing between male and female respondents. Table A6.1 presents a sample crosstab (ignore the \tilde{h} values in the table for the time being):

	Male	Female	Σ
Black	28	12	40
	$\tilde{h}_{11} = 20$	$\tilde{h}_{12} = 20$	
Silver	48	36	84
	$\tilde{h}_{21} = 42$	$\tilde{h}_{22} = 42$	
White	24	52	76
	$\tilde{h}_{31} = 38$	$\tilde{h}_{32} = 38$	
Σ	100	100	200

Table A6.1 Crosstab for χ^2 -test for independence

This 3x2 crosstab indicates that, in this case, 28 male respondents prefer the black smartphone, 48 the silver smartphone, and 24 the white smartphone. The last column (row) indicates the column (row) total indicated by the summation sign (Σ). In this sample, there are 100 males and 100 females. To answer the research question whether there is a relationship between the respondents' gender and their color preferences, we can apply the χ^2 -test for independence. Specifically, this tests the following hypothesis:

¹ This can also be done by using the Excel formula =CHIDIST(16.48,2). In the example, we have a χ^2 -statistic of 16.48 and 2 degrees of freedom, but the values can be modified.

H_0 : A color preference is independent of gender (and H_1 is: A color preference is dependent on gender)

Thus, this test examines the degree to which the observed frequencies deviate from the expected frequencies and is, therefore, similar to the one-sample χ^2 -test. The expected frequency of a cell \tilde{h}_{ij} (i being the index of the first variable with k categories and j being the index of the second variable with m categories) is the column total times the row total divided by the number of observations. This seems complicated, but is not. For example, the expected frequency of the cell male/black \tilde{h}_{11} is 100 (column total of the category “male”) times 40 (row total of the category “black”), divided by 200 (the total number of observations), which equals 20. Similarly, the expected frequency of the cell silver/male is $\tilde{h}_{21} = \frac{100 \cdot 84}{200} = 42$, and so on (see Table A6.1 for all cells’ expected frequencies).

An important property of the χ^2 -test is that it requires the expected frequency for each cell to be five or higher. In our example, this requirement is easily met. However, if this were not the case, we would have to apply Fisher’s exact test. For 2x2 tables, SPSS computes the Fisher’s exact test by default and we should interpret this test result instead of the χ^2 -test if there is a cell with an expected frequency of less than five (SPSS indicates this at the bottom of the Chi-Square Tests table). For tables with more than two rows or columns, the calculation of Fisher’s exact test becomes more complicated and requires running a Monte Carlo approach. By repeatedly sampling from a reference set of tables with the same dimensions, as well as row and column margins, as the observed table, the Monte Carlo approach obtains accurate Fisher’s test results. However, the Monte Carlo approach is only available if you have purchased SPSS Statistics Premium Edition or the Exact Tests

The computation of the χ^2 -test statistic is similar to the example above, the only exception being that we have to append a second summation sign as there are now two nominal variables:

$$\chi^2 = \sum_{i=1}^k \sum_{j=1}^m \frac{(h_{ij} - \tilde{h}_{ij})^2}{\tilde{h}_{ij}} = \frac{(28-20)^2}{20} + \dots + \frac{(52-38)^2}{38} = 18.43$$

The degrees of freedom are calculated as follows: $df = (k-1) \cdot (m-1)$. This example has 2 degrees of freedom and the associated critical value (for $\alpha = 0.05$) of 5.991 is much smaller than the test value (18.43). We can therefore assume that there is a significant relationship between the respondents’ gender and their preference for color. A problem associated with the χ^2 -test is that it only provides a weak indication of the strength of the association. Fortunately, there are several measures that help overcome this limitation; we introduce them in Box A6.1.

Box A6.1 Measures of the Strength of Association

There are several statistical measures that provide us with information regarding the strength of the association between nominal variables. Their computation only makes sense if the χ^2 -test or Fisher’s exact test renders significant results.

- The ϕ (*phi*) *coefficient* is used to measure the strength of association in 2x2 crosstabs. In fact, it is only a correlation coefficient for nominal variables and is computed as

follows:

$$\varphi = \sqrt{\frac{\chi^2}{n}}$$

If there is no association whatsoever between the variables (which is unrealistic), φ would be zero. Conversely, a value of 1 implies that the variables are perfectly associated. Generally, a φ value below 0.30 describes a weak association, between 0.30 and 0.49 a moderate one, and above 0.50 a strong association (Cohen 1992).

- The *contingency coefficient* (CC) is the equivalent to the φ coefficient for crosstabs with more than two rows and two columns. It is very similar to φ and also varies between 0 and 1, with higher values denoting a greater strength of association:

$$CC = \sqrt{\frac{\chi^2}{\chi^2 + n}}$$

CC is a very conservative measure and hardly ever reaches 1. Consequently, we should interpret CC in relation to its theoretical maximum value, which is defined as follows:

$CC_{max} = \sqrt{\frac{r-1}{r}}$, where r is the lesser of the number of rows and the number of columns. In our example, the number of columns (2) is smaller than the number of rows (3), and, consequently CC_{max} is 0.71. Using the information from our previous analysis, we can compute CC as follows:

$$CC = \sqrt{\frac{18.43}{18.43 + 200}} = 0.29$$

This value indicates that the association can be considered medium when compared to CC_{max} .

- *Cramer's V* is a modified version of the φ coefficient and is used in crosstabs larger than 2x2:

$$V = \sqrt{\frac{\chi^2}{n \cdot (r - 1)}} = \sqrt{\frac{18.43}{200 \cdot (2 - 1)}} = 0.30$$

Other measures include the *lambda*, *uncertainty coefficient*, and *Tshuprow's T*. While all these measures are used to measure the strength of association between nominal variables, SPSS provides different statistics, such as *Kendall's tau-b* (pronounced as *tau*), *Kendall's tau-c*, *Somer's d*, and γ (pronounced as *gamma*) for ordinal variables. Essentially, these measures use information on the ordering of variables' categories by considering every possible pair of cases in the crosstab. They vary between -1 and +1 and thus distinguish between positive and negative relationships. Higher absolute values denote a stronger degree of association. For more information, see, for example, Fleiss et al. (2003).

Table A6.2 provides an overview of the steps involved when carrying out the following nonparametric tests in SPSS: One-sample χ^2 -test, and the χ^2 -test for independence. Compare Chapter 6 of the book for a description of the Kolmogorov-Smirnov and the Shapiro-Wilk tests.

Theory	Action
<i>One-sample χ^2-test</i>	
Compare the proportions of cases that fall within the various categories of a nominal or ordinal variable with a given standard	Go to ► Analyze ► Nonparametric Tests ► Legacy Dialogs ► Chi-square
<i>Assumptions</i>	
Is the test variable measured on a nominal or ordinal scale?	Check Chapter 3 to determine the measurement level of your variables
Are the observations independent?	Consult Chapter 3 to determine if the observations are independent
<i>Specification</i>	
Select the test variable	Enter the variable into the Test Variable List box
Select the expected values	If the expected values are uniformly distributed across the categories, no action is necessary; otherwise, pre-specify the expected values in the Expected Values box
<i>Results interpretation</i>	
Examine the test results	Examine the χ^2 -value and its significance level
<i>χ^2-test for independent samples</i>	
Determine whether two nominal or ordinal variables are related	Go to ► Analyze ► Descriptive Statistics ► Crosstabs
<i>Assumptions</i>	
Is the test variable measured on a nominal or ordinal scale?	Check Chapter 3 to determine the measurement level of your variables
Are the observations independent?	Consult Chapter 3 to determine if the observations are independent
<i>Specification</i>	
Select the test variables	Enter the crosstab's row and column variables into the Row(s) and Column(s) boxes
Select the test statistic	Go to ► Analyze ► Descriptive Statistics ► Crosstabs ► Statistics. Select Chi-square . If one of the cells has an expected frequency of less than five, go to ► Analyze ► Descriptive Statistics ► Crosstabs ► Exact and click on Monte Carlo . Maintain the default settings.
Select the measures of the strength of association	Go to ► Analyze ► Descriptive Statistics ► Crosstabs ► Statistics and choose measures according to the scale level (nominal or ordinal)
<i>Results interpretation</i>	
Examine the test results	

Determine the strength of the effects	If the expected frequency is five, or more, for all cells, examine the χ^2 -value and its significance level; if the expected frequency is less than five for one cell, examine Fisher's exact test results.
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Examine the measures of the strength of association

Table A6.2 Steps involved in carrying out selected parametric and nonparametric tests in SPSS

Reference

Cohen, J. (1992). A power primer. *Psychological Bulletin*, 112(1), 155–159.

Fleiss, J. L., Levin, B., & Paik, M. C. (2003). *Statistical methods for raters and proportions* (3rd ed.), New York et al.: Wiley.

IBM (2011). *One-Sample Chi-Square Test (nonparametric tests algorithms)* on [https://www.ibm.com/support/knowledgecenter/pl/SSLVMB_20.0.0/com.ibm.spss.statistics.h
elp/alg_npar_tests_onesample.htm](https://www.ibm.com/support/knowledgecenter/pl/SSLVMB_20.0.0/com.ibm.spss.statistics.help/alg_npar_tests_onesample.htm)